BIRZEIT UNIVERSITY

Faculty of Engineering and Technology
Department of Electrical and Computer Engineering
Engineering Probability and Statistics ENEE 2307

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Midterm Exam

Date: Saturday July 28, 2018 Time: 75 minutes
Name: Student #:

Opening Remarks:

- This is a 75-minute exam. Calculators are allowed. However, books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem 1: 20 Points

Given three independent events A, B, and C such that P(A) =0.6, P(B) =0.5, and P(C) = 0.4, find the probability that

a. At least one event occurs
b. All three events occur.
c. Both of A and B occur.

Problem 2: 20 Points

A box contains three coins A, B and C. Coins A and B are two headed, while coin C is a fair one (has a head H and a tail T). One coin is chosen at random from the box and the tossed once.

a. What is the probability that the toss results in a head H?
b. If the picked coin shows a heads H, find the probability that coin C was selected, i.e., the fair coin?

Problem 3: 20 Points

The waiting time, X, in minutes, between successive speeders (المتجاوزين للسرعة) spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F_X(x) = \begin{Bmatrix} 0 & x < 0 \\ 1 - e^{-2x} & x \geq 0 \end{Bmatrix}$$

a. Find the probability density function $f_X(x)$
b. Find the probability that the waiting time between successive speeders is less than 2 minutes.
c. What is the average waiting time, in minutes, between successive speeders?

Problem 4: 20 Points

In testing a certain kind of truck tires, it is found that 5 % of the tires fail to complete the test run without a blowout (ينفجر).

a. Find the probability that out of 20 tested tires at least two have blowouts.
b. How many of the 20 tested tires would you expect to have blowouts?

Problem 5: 20 Points

Suppose that the proportion of colorblind (عمى الالوان) people in a large population is 0.005. Use the normal approximation to calculate the probability that there will be at most 32 colorblind persons in a randomly chosen group of 6000 people.

Good Luck

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Problem 1: $P(A) = 0.6$, $P(B) = 0.5$, $P(C) = 0.4$

a. $P(\text{at least one event}) = P(A \cup B \cup C)$

$= P(A) + P(B) + P(C)$

$- P(A)P(B) - P(A)P(C) - P(B)P(C)$ } due to independence

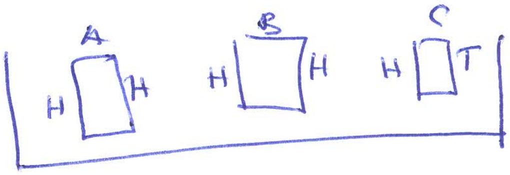
$+ P(A)P(B)P(C)$

$= 1.5 - 0.3 - 0.24 - 0.2 + 0.12 = 0.88$ 8

b. $P(A \cap B \cap C) = (0.6)(0.5)(0.4) = 0.12$ 6

c. $P(A \cap B) = (0.6)(0.5) = 0.3$ 6

Problem 2



$P(H) = P(A)P(H/A) + P(B)P(H/B) + P(C)P(H/C)$

a. $= \frac{1}{3}(1) + \frac{1}{3}(1) + \frac{1}{3}\left(\frac{1}{2}\right)$

$P(H) = \frac{5}{6}$ (12)

b. $P(C/H) = \frac{P(C \cap H)}{P(H)} = \frac{P(C)\,P(H/C)}{P(H)}$

$= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{5}{6}} = \frac{1}{5}$ (8)

Problem 3 : $F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-2x} & x \geq 0 \end{cases}$

a. $f_X(x) = \dfrac{d F_X(x)}{dx} = \begin{cases} 0 & x < 0 \\ 2 e^{-2x} & x \geq 0 \end{cases}$ 7

b. $P(X \leq 2) = \int_0^2 2 e^{-2x} \, dx = -e^{-2x} \Big|_0^2$

$P(X \leq 2) = 1 - e^{-4}$ 7

c. $E(X) = \int_0^\infty 2x \, e^{-2x} \, dx = \int_0^\infty x f_X(x) \, dx =$

$= \dfrac{1}{\lambda} = \dfrac{1}{2}$ 6

Problem 4 :

$n = 20$

$p = 0.05$

$P(X = x) = \binom{20}{x} (0.05)^x (0.95)^{20-x}$ 6

a. $P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$ 3

$= 1 - \left[\binom{20}{0} (0.05)^0 (0.95)^{20} + \binom{20}{1} (0.05)^1 (0.95)^{19} \right]$ 3

$= 1 - [0.3584 + 0.3773]$

$= 0.2642$ 2

b. $E(X) = np$

$= (20)(0.05)$

$= 1$

$\dfrac{6}{20}$

Problem 5

$p = 0.005$

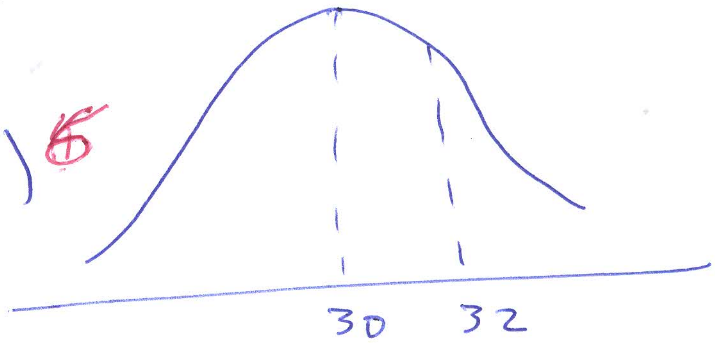
$n = 6000$

Find $P(X \le 32)$

$\mu_x = np = 6000 * 0.005 = 30$ 5

$\sigma_x^2 = np(1-p) = 6000 * 0.005 * (1 - 0.005) = 29.85$

$\sigma_x = 5.463$ 5



$P(X \le 32) = \Phi\left(\frac{32 - \mu_x}{\sigma_x}\right)$ 5

$= \Phi\left(\frac{32 - 30}{5.463}\right)$ 2

$= \Phi(0.366)$

$= 0.6406$ 2 (0.6443)